c=area $(KUP+OVN\times MSR+QTL)=(152-48\surd 2)$ sq. inches.

$\therefore\quad p$=chance that it touches or falls on both diagonals$=\dfrac{a}{A}=\dfrac{1}{144}$.

$\therefore\quad p_1$=chance that it touches or falls on one diagonal$=\dfrac{b}{A}=\dfrac{48\surd 2-9}{144}$.

$\therefore\quad p_2$=chance that it does not touch or fall on a diagonal$=\dfrac{c}{A}=\dfrac{152-48\surd 2}{144}$.

$\therefore\quad p+p_1+p_2=1$.

NOTE.—This problem was solved with different results by H. W. Draughon, Hon. Sosiah Drummond and ———. Professor Draughon and ———'s result is $\frac{1}{121}$. They suppose that the surface of the coin must be entirly on the given square, thus reducing the area of the surface upon which the centre of the coin may fall by a half-inch strip on each side.

Mr. D. consider the surface as though it were the bottom of a box. In this case, the area on which the coin could fall is $\left[144-\left(1-\frac{\pi}{4}\right)\right]$ square inches. Then the probability required is $\dfrac{4}{572-\pi}$. Each of these three results is right when viewed from the stand-point of its author, but we are doubtful whether the result $\frac{1}{121}$ can be harmonized with the theory of probability.

11. Proposed by ARTEMAS MARTIN, A. M., Ph. D., LL. D., U. S. Coast and Geodetic Survey Office, Washington, D. C.

Find the average area of a triangle formed by joining a corner of a cube with any two points within the cube.

Solution by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland, and Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

Take a lower corner O of the cube as the origin of co-ordinates, and let P_1 and P_2 be any two points taken at random within the cube. Make (x, y, z) and (u, v, w) the Cartesian triple co-ordinates of P_1 and P_2 respectively; then will $OP_1=\surd(x^2+y^2+z^2)$, and $OP_2=\surd(u^2+v^2+w^2)$.

Considering P_1 the *remoter* point with respect to the origin of co-ordinates, we have $P_1P_2=\surd[(x-u)^2+(y-v)^2+(z-w)^2]$; and consequently,

$$\cos\angle P_1OP_2=\cos\phi=\frac{(OP_1)^2-(OP_2)^2-(P_1P_2)^2}{2(OP_1)(OP_2)},=\frac{ux+vy+wz}{2(OP_1)(OP_2)}.$$

$$\therefore\quad \sin\phi=\surd(1-\cos^2\phi)=\frac{\surd[(vx-uy)^2+(wx-uz)^2+(wy-vz)^2]}{2(OP_1)(OP_2)},\ \text{and}$$

$$\triangle P_1OP_2=A=\tfrac{1}{2}(OP_1)(OP_2)\sin\phi=\tfrac{1}{4}\surd[(vx-uy)^2+(wx-uz)^2+(wy-vz)^2].$$

Hence the required average area becomes

$$\mathbf{A}=\frac{\int_0^a\int_0^a\int_0^a\int_0^x\int_0^y\int_0^z A\,dxdydzdudvdw}{\int_0^a\int_0^a\int_0^a\int_0^x\int_0^y\int_0^z dxdydzdudvdw},$$

in which x represents a side of the given cube. The labor required to perform the indicated integrations is *enormous*—enough to discourage the most enthusiastic mathematical genius.

NOTE—Since the parenthetical expressions in $\triangle P_1P_2O=\frac{1}{2}\sqrt{[(vx-uy)^2+(wx-uz)^2+(wy-vz)^2]}$ represent respectively 2(Area of the projections of of $\triangle OP_2P_1$) on the co ordinate planes XY, ZX, the result of problem 2, Average and Probability in April No. of the MONTHLY, substituted in these expressions gives $\triangle P_1P_2O=\frac{13}{108}a^2\sqrt{3}$ as the required average area.—MATZ.

MISCELLANEOUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

11. Proposed by CHARLES E. MYERS, Canton Ohio

"Assuming the earth's orbit to be a circle, if a comet move in a parabola around the sun and in the plane of the earth's orbit, show that the comet cannot remain within the earth's orbit longer than 78 days."

I. Solution by WILLIAM HOOVER, A. M. Ph. D., Professor of Mathematics and Astronomy in the Ohio University, Athens, Ohio

Let $4a$ = the latus rectum of the comet's orbit, ρ=its distance at any time from the sun, p=the perpendicular from the sun upon a tangent to the comet's path, F=the attractive force of the sun, h=the double area described by ρ in a unit of time, and θ the angular co-ordinate corresponding to ρ. Then $F=\frac{h^2}{p^3}\frac{dp}{d\rho}$ ....(1), and from the parabola $\frac{2}{p^3}\frac{dp}{d\rho}=\frac{1}{a\rho^2}$; $\therefore$ $F=\frac{h^2}{2a\rho^2}$....(2).

Let $F=\phi$, when $\rho=1$; then $h=\sqrt{(2a\phi)}$....(3), and $F=\frac{\phi}{\rho^2}$....(4).

If r=the radius of the earth's orbit, and v=the velocity of the earth, $v^2=rF=\frac{\phi}{r}$, or $v=\sqrt{\frac{\phi}{r}}$. Then $\theta r\div v=\frac{\theta(r)^{\frac{3}{2}}}{\sqrt{\phi}}$ =the time required for the earth to describe the arc subtending the angle θ at the sun....(e).

For the comet, $dt=\frac{\rho^2d\theta}{h}$ ....(5), and from the parabola,

$\rho=\frac{2a}{1+\cos\theta}=\frac{a}{\cos^2\frac{1}{2}\theta}$....(6). This gives $\rho^2d\theta=\frac{a^2d\theta}{\cos^4\frac{1}{2}\theta}$, and then (5) gives

$$t=\frac{2a^2}{h}\int\frac{d\theta}{\cos^4\frac{1}{2}\theta}=\frac{2a^2}{\sqrt{2a\phi}}\left\{\tan\tfrac{1}{2}\theta+\tfrac{1}{3}\tan^3\tfrac{1}{2}\theta\right\}\text{....(7)}.$$